```
eq1 = (e^(-x (1+f12)-y (1+f21)) ((-y + e^(x f12+y f21) (x + y)) f12 + y f21)) /
      ((x + y) (x f12 + y f21)) == (e^(-x-y) (-1 + e^x)) / ((-1 + e^(x+y)) x)
```

$$\frac{e^{-x\,(1+f_{12})-y\,(1+f_{21})}\left(\left(-y+e^{x\,f_{12}+y\,f_{21}}\,(x+y)\right)\,f_{12}+y\,f_{21}\right)}{(x+y)\,(x\,f_{12}+y\,f_{21})}==\frac{e^{-x-y}\,(-1+e^{x})}{(-1+e^{x+y})\,x}$$

```
eq2 = (e^(-x-x f12-y f21) (x y (f12 - f21) + e^(x f12+y f21) y (x + y) f21 + e^((x+y)/2) x (x f12 + y f21))) /
      ((x + y) (x f12 + y f21)) == (e^(-x g12-y g21) x g12 + y g21) / (x g12 + y g21)
```

$$\left(e^{-x-x\,f_{12}-y\,f_{21}}\left(x\,y\,(f_{12}-f_{21})+e^{x\,f_{12}+y\,f_{21}}\,y\,(x+y)\,f_{21}+e^{\frac{x+y}{2}}\,x\,(x\,f_{12}+y\,f_{21})\right)\right)/((x+y)\,(x\,f_{12}+y\,f_{21}))==\frac{e^{-x\,g_{12}-y\,g_{21}}\,x\,g_{12}+y\,g_{21}}{x\,g_{12}+y\,g_{21}}$$

```
Simplify[eq1 /. (a_ == b_) :> (a / b)]
```

$$\left(e^{-x\,f_{12}-y\,f_{21}}\,(-1+e^{x+y})\,x\left(\left(-y+e^{x\,f_{12}+y\,f_{21}}\,(x+y)\right)\,f_{12}+y\,f_{21}\right)\right)/((-1+e^{x})\,(x+y)\,(x\,f_{12}+y\,f_{21}))$$

```
t0 = Simplify[
  eq1 /. (a_ == b_) :> (a / b) /. {f12 -> (g[x, y] - 1 / 4) / x, f21 -> (g[y, x] + 1 / 4) / y}]
```

$$\left(e^{-g[x,y]-g[y,x]}\,(-1+e^{x+y})\left(-\left(-1+e^{g[x,y]+g[y,x]}\right)\,(x+y)+4\left(-y+e^{g[x,y]+g[y,x]}\,(x+y)\right)\,g[x,\,y]+4\,x\,g[y,\,x]\right)\right)/(4\,(-1+e^{x})\,(x+y)\,(g[x,\,y]+g[y,\,x]))$$

```
t1 = Simplify[eq1 /. (a_ == b_) :> (a / b) /.
      {f12 -> (g[x, y] - 1 / 4) / x, f21 -> (g[y, x] + 1 / 4) / y} /. y -> x /. g[x, x] -> g]
```

$$\frac{e^{-2\,g}\,(1+e^{x})\left(1+e^{2\,g}\,(-1+4\,g)\right)}{8\,g}$$

```
Solve[t1 == 1, g]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{g\to\left(1+e^{x}-2\,\text{ProductLog}\left[-\frac{e^{\frac{2\,(-1-e^{x})}{-4+4\,e^{x}}}\,(1+e^{x})}{2\,(-1+e^{x})}\right]+2\,e^{x}\,\text{ProductLog}\left[-\frac{e^{\frac{2\,(-1-e^{x})}{-4+4\,e^{x}}}\,(1+e^{x})}{2\,(-1+e^{x})}\right]\right)/(4\,(-1+e^{x}))\right\}\right\}$$

```
Simplify[eq1 /. (a_ == b_) :> (a / b) /.
    {f12 -> (g[x, y] - 1 / 4) / x, f21 -> (g[y, x] + 1 / 4) / y} /. y -> -x]
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. >>

Infinity::indet : Indeterminate expression
$\left(0\,e^{x\left(1-\frac{\ll 1\gg+\ll 1\gg}{x}\right)-x\left(1+\frac{\ll 1\gg}{x}\right)}\,\text{x ComplexInfinity}\,(g[-x,\,x]+g[x,\,-x])\right)\Big/((-1+e^{x})\,(g[-x,\,x]+g[x,\,-x]))$ encountered. >>

```
Indeterminate
```

```
Simplify[t0 /. {x → z x, y → x / z}]
```

$$\left(e^{-g\left[\frac{x}{z},\,x\,z\right]-g\left[x\,z,\,\frac{x}{z}\right]}\left(-1+e^{x\left(\frac{1}{z}+z\right)}\right)\left(-\left(-1+e^{g\left[\frac{x}{z},\,x\,z\right]+g\left[x\,z,\,\frac{x}{z}\right]}\right)\left(1+z^2\right)+4\,z^2\,g\left[\frac{x}{z},\,x\,z\right]+4\left(-1+e^{g\left[\frac{x}{z},\,x\,z\right]+g\left[x\,z,\,\frac{x}{z}\right]}\right)\left(1+z^2\right)\right)g\left[x\,z,\,\frac{x}{z}\right]\right)\Big/\left(4\left(-1+e^{x\,z}\right)\left(1+z^2\right)\left(g\left[\frac{x}{z},\,x\,z\right]+g\left[x\,z,\,\frac{x}{z}\right]\right)\right)$$

```
t0
```

$$\left(e^{-g[x,y]-g[y,x]}\left(-1+e^{x+y}\right)\left(-\left(-1+e^{g[x,y]+g[y,x]}\right)(x+y)+4\left(-y+e^{g[x,y]+g[y,x]}(x+y)\right)g[x,\,y]+4\,x\,g[y,\,x]\right)\right)\Big/\left(4\left(-1+e^{x}\right)(x+y)\left(g[x,\,y]+g[y,\,x]\right)\right)$$

```
Solve[a F + b F E^F + c E^(2 F) + d E^F + e == 0, F]
```

Solve::nsmet : This system cannot be solved with the methods available to Solve. ≫

```
Solve[e + d e^F + c e^(2 F) + a F + b e^F F == 0, F]
```

```
Solve[a F + b F E^F + 0 E^(2 F) + d E^F + e == 0, F]
```

Solve::nsmet : This system cannot be solved with the methods available to Solve. ≫

```
Solve[e + d e^F + a F + b e^F F == 0, F]
```

```
Solve[a F + b F E^F + d E^F + e == 0, F]
```

Solve::nsmet : This system cannot be solved with the methods available to Solve. ≫

```
Solve[e + d e^F + a F + b e^F F == 0, F]
```

```
Solve[a F + b F E^F + d E^F == 0, F]
```

Solve::nsmet : This system cannot be solved with the methods available to Solve. ≫

```
Solve[d e^F + a F + b e^F F == 0, F]
```

```
Solve[a F + b E^F + c == 0, F]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{F\to\frac{-c-a\,\text{ProductLog}\left[\frac{b\,e^{-\frac{c}{a}}}{a}\right]}{a}\right\}\right\}$$

```
t1
```

$$\frac{e^{-2g}\left(1+e^{x}\right)\left(1+e^{2g}\left(-1+4g\right)\right)}{8g}$$

```
Expand[8 g (t1 - 1)]
```

$$-1+e^{-2g}-e^{x}+e^{-2g+x}-4g+4e^{x}g$$